8. L. V. Biryukova, N. N. Danilov, and E. N. Sinitsyn, "Superheating of pure liquids and binary solutions with convective heat exchange with a thin wire," in: Thermophysics of Metastable Systems [in Russian], Ural Science Center, Academy of Sciences of the USSR, Sverdlovsk (1977), pp. 16-22.
9. V. I. Tolubinskii, Yu. N. Ostrovskii, and V. E. Pisarev, "Delay of boiling in nonsteady heat transfer with phase transformations," Teplofiz. Teplotekh., 34, 57-60 (1978).
10. B. P. Avksentyuk and S. S. Kutateladze, "Instability of the heat transfer regime on surfaces depleted of centers of vaporization," Teplofiz. Vys. Temp., 15, No. 1, 115-120 (1977).
11. B. V. Artem'ev, M. N. Ivanovskii, L. A. Litvinova, and I. M. Sviridenko, "Regime of impeded heat transfer of an abruptly heated wire in a large volume of liquid," Teplofiz. Vys. Temp., 17, No. 6, 1259-1264 (1979).
12. D. N. Sinha, L. C. Brodie, J. S. Semura, and F. M. Young, "Premature transition to stable film boiling initiated by power transients in liquid nitrogen," Cryogenics, 19, No. 4, 225-230 (1979).
13. O. E. Dwyer, Boiling Liquid-Metal Heat Transfer, American Nuclear Society (1976).

ENTRANCE SECTION OF A PLANAR TURBULENT JET IN TRANSVERSE FLOW

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The author presents a solution for the entrance section of a planar turbulent jet in a carrier stream, accounting for additional ejection of liquid in the upstream mixing zone.

Reference [1] examined a solution of the problem of the entrance section of a planar turbulent jet generated in a transverse flow, using an integral method. The system of jet boundary layer equations, written in curvilinear coordinates fixed to the jet axis, is closed by means of the Prandtl formula for the shear stresses in which curvature is not accounted for. For high carrier stream speeds this solution does not agree well with experimental data.

Figure 1 shows the experimentally obtained width of the mixing zone, as described in [1], for jet to flow velocity ratios of $u_0/V_\infty = 9.35, 4.83, 3.23$. The broken lines are the width of the mixing zone for the ordinary immersed jet ($V_\infty = 0$). It can be seen from Fig. 1 that the upstream mixing zone is wider than the downstream one, and the difference between them increases with increase of the carrier stream velocity. The jet in the upstream mixing zone seems to eject additional mass, compared with the ejection into the mixing zone of the ordinary immersed jet and into the downstream mixing zone. The apparent cause is the influence of jet curvature, which can be quite large in the entrance section, on the intensity of mixing. It is known (see, e.g., [2]) that mixing proceeds with greater intensity in a stream flowing along a curved convex wall where the velocity falls with increasing distance from the wall, since centrifugal force ejects fast particles along the radius from the center of curvature with greater intensity than slow particles, and therefore the thickness of the mixing zone must be larger than when there are no centrifugal forces. The presence of a centrifugal force is linked to curvature of the jet, and the centrifugal force is greater, the greater is the component of carrier stream velocity normal to the axis. An increase of the speed of the carrier stream for a given initial jet speed leads to an increase of jet curvature, of the carrier stream velocity component normal to its axis, and of the centrifugal force. This in turn must lead to an increase in the additional mass coupled to the forward part of the jet. The jet curves with increasing distance form the source, the angle between the axis and the carrier stream is reduced, and therefore the normal flow velocity component is reduced and tends to zero in the limit. Therefore, the added ejection should not be taken into account in calculating the main section. But in the development of the entrance section, the added ejection into the upstream mixing zone of the jet must play an important role.

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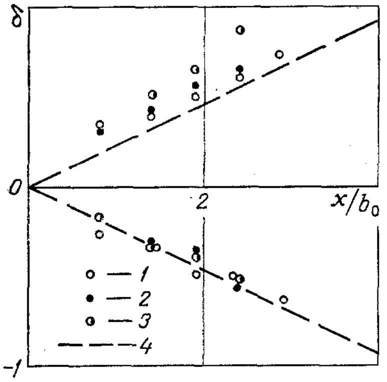

Fig. 1

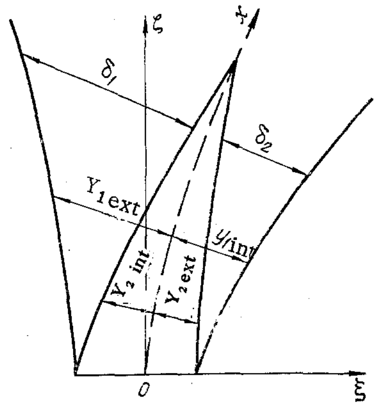

Fig. 2

Fig. 1. The width of the upstream and downstream mixing zones as a function of the distance from the nozzle rim: 1) $u_0/V_\infty = 9.35$; 2) 4.83; 3) 3.23; 4) width of the mixing zone of an immersed jet.

Fig. 2. Diagram of the entrance section of a planar jet in a carrier stream.

Below we present the solution of the problem of the entrance section of a planar turbulent jet in a carrier stream, accounting for the added ejection in the upstream part of the jet. To close the system of boundary layer equations, written in curvilinear coordinates fixed to the jet axis, following [3] we use the differential equation for the mass flux of liquid ejected by the jet.

To solve the problem we make the following assumptions: 1) The curved jet axis is a zero stream line; 2) the radius of curvature of the curved axis in the entrance section is constant; 3) in the constant total pressure core the transverse velocity is considerably less than the longitudinal; 4) the variation of mass flow rate in the internal (downstream) mixing zone is proportional to the velocity at the interior boundary of the core; 5) the variation of excess mass flow rate in the external (upstream) mixing zone is equal to the sum of two components, proportional, respectively, to the excess velocity in the jet core (relative to the velocity at the upstream boundary of the jet) and to the normal velocity component of the carrier stream; 6) in each of the mixing zones the velocity profiles are similar across the mixing zone; and 7) in accordance with [4] we can consider the pressure behind the jet to be $2(p_\infty - p_0)/V_\infty^2 \simeq 0.7$.

It should be noted that reference [5] also used a differential equation for the mass flow rate of liquid ejected by the jet, but there the flow rate was considered to be proportional only to the difference in the longitudinal components of velocity in the jet core and in the stream, i.e., the added ejection was not accounted for.

The conditions at the external boundary, as was true in [1], are determined from considering flow over the jet as boundary-free potential flow over a solid curved wall. Here, proceeding from assumption 2, the velocity and pressure at the external jet boundary are determined from the boundary-free zero-circulation flow with velocity V_∞ over a cylinder of radius R. At the internal boundary the velocity and pressure were taken as zero (the pressure was computed from the static pressure behind the jet p_0).

The velocities in the potential core were determined in [1] and have the form (Fig. 2)

$$u_1 = u_0 \exp[-(b_{02} + y)/R]. \tag{1}$$

To determine the boundaries of the internal mixing zone we use the condition of conservation of momentum of the internal part of the jet and the equation for the variation of mass flux through the jet

$$\frac{d}{dx} \int\limits_{y_{1\,\text{int}}}^{0} (u^2 + p/\rho)\, dy = 0, \tag{2}$$

$$\frac{d}{dx}\int_{y_{1\,\mathrm{int}}}^{0} u\,dy = v_e = c_1 u_{1\,\mathrm{int}}. \tag{3}$$

Here u is the longitudinal velocity component (along the curved axis) of the liquid in the jet; ρ, density; v_e, transverse velocity of liquid ejected by the jet; and c_1, empirical constant.

Correspondingly, to determine the boundaries of the external mixing zone we can use an integral relation for the excess jet momentum and the equation for variation of the excess mass flux of impurity along the jet axis.

The first relation is obtained by integrating the equation of motion, transformed with the aid of the continuity equation, across the jet from y = 0 to $y = y_{1\mathrm{ext}}$, and has the form

$$\frac{d}{dx}\int_{0}^{y_{1\,\mathrm{ext}}} [u(u - u_\delta) + p/\rho]\,dy = y'_{1\,\mathrm{ext}}\frac{p_\delta}{\rho} - \frac{du_\delta}{dx}\int_{0}^{y_{1\,\mathrm{ext}}} u\,dy. \tag{4}$$

According to assumption 5, the equation for the excess mass flux of impurity has the form

$$\frac{d}{dx}\int_{0}^{y_{1\,\mathrm{ext}}} (u - u_\delta)\,dy = c_1(u_{1\,\mathrm{ext}} - u_\delta) + c_2 V_\infty \cos\alpha. \tag{5}$$

Here c_2 is a second empirical constant.

According to assumption 6, the dimensionless velocity profiles in the internal and external mixing zones can be represented in the form

$$\frac{u}{u_{1\,\mathrm{int}}} = \begin{cases} 2\eta_{\mathrm{int}}^{3/2} - \eta_{\mathrm{int}}^{3} & (0 \leqslant \eta_{\mathrm{int}} \leqslant 1) \\ 0 & (\eta_{\mathrm{in}} > 1) \end{cases}, \quad \frac{u - u_\delta}{u_{1\mathrm{ext}} - u_\delta} = \begin{cases} 2\eta_{\mathrm{ext}}^{3/2} - \eta_{\mathrm{ext}}^{3} & (0 \leqslant \eta_{\mathrm{ext}} \leqslant 1) \\ 0 & (\eta_{\mathrm{ext}} > 1) \end{cases}, \tag{6}$$

where

$$\eta_{\mathrm{int}} = \frac{y_{1\mathrm{int}} - y}{y_{1\mathrm{int}} - y_{2\,\mathrm{int}}}, \quad \eta_{\mathrm{ext}} = \frac{y_{1\mathrm{ext}} - y}{y_{1\,\mathrm{ext}} - y_{2\,\mathrm{ext}}}. \tag{7}$$

For the integration within the jet core we use an expression, Eq. (1), for the velocity, and we determine the pressure in the mixing zone by integrating the condition for transverse equilibrium across the jet (see [4, 6])

$$\frac{1}{\rho}\frac{\partial p}{\partial y} = \frac{u^2}{R}, \tag{8}$$

whence in the internal mixing zone we obtain

$$\int_0^1 \frac{p}{\rho}\,d\eta_{\mathrm{int}} = -0{,}106\,\frac{\delta_2}{R}\,u_{1\,\mathrm{int}}^2 \tag{9}$$

and in the external mixing zone we obtain

$$\int_0^1 \frac{p}{\rho}\,d\eta_{\mathrm{ext}} = \frac{\delta_1}{R}\left\{-0{,}416\delta_2 u_0^2 + \frac{R}{2}(u_{1\mathrm{int}}^2 - u_{1\,\mathrm{ext}}^2) + \delta_1\left[\frac{1}{2}u_\delta^2 + 0{,}744 u_\delta(u_{1\mathrm{ext}} - u_\delta) + 0{,}310(u_{1\,\mathrm{ext}} - u_\delta)^2\right]\right\}. \tag{10}$$

Following integration of Eqs. (2)-(5) under the conditions

$$\delta_1 = \delta_2 = 0,\ y_{2\,\mathrm{int}} = -b_{02},\ y_{2\mathrm{ext}} = b_{01} \quad \text{at} \quad x = 0 \tag{11}$$

and after writing these equations in a form allowing successive approximations to be constructed in order to determine δ_2, δ_1, y_{2int} and y_{2ext}, we obtain

$$\delta_2 = -\frac{u_0}{0.55u_{1\,int}}\left[c_1\int_0^1 \frac{u_{1\,int}}{u_0}dx + R\left(1-\frac{u_{1\,int}}{u_0}\right)\right], \tag{12}$$

$$\delta_1 = \frac{1}{0{,}55\left(\frac{u_{1\,ext}}{u_0}-\frac{u_\delta}{V_\infty}\frac{V_\infty}{u_0}\right)}\left\{c_1\int_0^x \frac{u_{1ext}}{u_0}dx - 2c_1\frac{V_\infty}{u_0}\xi + c_2\frac{V_\infty}{u_0}\zeta + \frac{u_\delta}{V_\infty}\frac{V_\infty}{u_0}y_{2ext} - R\left[\exp\left(-\frac{1}{R}\right)-\frac{u_{1ext}}{u_0}\right]\right\}, \tag{13}$$

$$y_{2int} = -b_{02} - \frac{R}{2}\left(1-\frac{u_{1\,int}^2}{u_0^2}\right) - 2\delta_2\left(0{,}416 - 0{,}106\frac{\delta_2}{R}\right)\frac{u_{1\,int}^2}{u_0^2}, \tag{14}$$

$$y_{2ext} = b_{01}\left(1-\frac{V_\infty^2}{u_0^2}\right) + \frac{R}{2}\left[\exp\left(-\frac{2b_{02}}{R}\right) - \exp\left(-\frac{2}{R}\right)\right] + \frac{2p_\delta}{\rho V_\infty^2}\left(\frac{V_\infty}{u_0}\right)^2\left(y_{2ext}+\delta\right) - \frac{V_\infty^2}{u_0^2}\int_0^x y_{1ext}\frac{d\left(\frac{2p_\delta}{\rho V_\infty^2}\right)}{dx}dx -$$

$$-2\int_0^x \frac{du_\delta/V_\infty}{dx}\frac{V_\infty}{u_0}\left\{R\left[\exp\left(-\frac{b_{02}}{R}\right) - \frac{u_{1\,ext}}{u_0}\right] + \delta_1\left(0{,}45\frac{u_\delta V_\infty}{V_\infty u_0} + 0{,}55\frac{u_{1ext}}{u_0}\right)\right\}dx - \frac{R}{2}\left\{\exp\left(-\frac{2b_{02}}{R}\right) - \frac{u_{1\,ext}^2}{u_0^2} -\right.$$

$$\left.-4\frac{u_\delta V_\infty}{V_\infty u_0}\left[\exp\left(-\frac{b_{02}}{R}\right) - \frac{u_{1\,ext}}{u_0}\right]\right\} - 2\delta_1\left(\frac{u_{1ext}}{u_0} - \frac{u_\delta V_\infty}{V_\infty u_0}\right)\times$$

$$\times\left(0{,}134\frac{u_\delta V_\infty}{V_\infty u_0} + 0{,}416\frac{u_{1ext}}{u_0}\right) - 2\frac{\delta_2}{R}\left[-0{,}416\delta_2 + \frac{R}{2}\left(\frac{u_{1\,int}^2}{u_0^2} -\right.\right.$$

$$\left.\left.-\frac{u_{1\,ext}^2}{u_0^2}\right) + \delta_1\left(0{,}066\frac{u_\delta^2 V_\infty^2}{V_\infty^2 u_0^2} + 0{,}124\frac{u_{1ext}u_\delta V_\infty}{u_0 V_\infty u_0} + 0{,}310\frac{u_{1ext}^2}{u_0^2}\right)\right]. \tag{15}$$

In the limit as $V_\infty \to 0$ and $R \to \infty$, from Eqs. (12)-(15) we obtain a solution for the ordinary immersed jet

$$\delta = \frac{c_1}{0{,}134}x, \quad y_2 = 0.5 - 0{,}416\delta. \tag{16}$$

From the first equation of Eq. (16) we can evaluate the empirical constant c_1. Since $\delta_1 \simeq 0.3$ for the immersed jet, we have

$$c_1 \sim 0{,}04. \tag{17}$$

Equations (12)-(15) can be solved simultaneously by the method of successive approximations. These equations contain the quantities u_δ/V_∞, $2p_\delta/\rho V_\infty^2$, and R, the radius of curvature of the jet axis. The first two equations can be found, as was noted above, as the velocity and pressure on the surface of a cylinder of radius R washed by a stream with velocity V_∞:

$$\frac{u_\delta}{V_\infty} = 2\sin\frac{x}{R}, \tag{18}$$

$$\frac{2p_\delta}{\rho V_\infty^2} = 1{,}7 - 4\sin^2\frac{x}{R}. \tag{19}$$

Here

$$x = R \arcsin \frac{\zeta}{R} \tag{20}$$

and we note that the static pressure is calculated from the pressure in the reverse flow zone behind the jet, which, in accordance with assumption 7, is taken to be $p_0 = p_\infty - 0.7\rho \frac{V_\infty^2}{2}$. Here p_0 is the pressure in the reverse flow zone, and p_∞ is the pressure in the flow ahead of the jet at a large distance from it.

The radius of curvature was determined from the condition that at the nozzle lip the pressure at the external boundary of the core is the stagnation pressure of the carrier stream (allowing for expansion behind the jet)

$$1.7 \frac{V_\infty^2}{2} = \frac{u_0^2}{2} - \frac{u_{1\,\mathrm{ext}}^2}{2}$$

or

$$\frac{u_0^2 - 1.7V_\infty^2}{u_0^2} = \exp\left(-\frac{2}{R}\right),$$

whence

$$R = 2/\ln\left(\frac{u_0^2/V_\infty^2}{u_0^2/V_\infty^2 - 1.7}\right). \tag{21}$$

The quantities $y_{2\mathrm{int}}$, $y_{2\mathrm{ext}}$, δ_2 and δ_1 were calculated in a computer. Here, besides these quantities, for each value of u_0/V_∞ we calculated the position of the jet axis at the nozzle lip relative to its trailing edge (b_{02}), from the condition that the core boundaries $y_{2\mathrm{int}}$ and $y_{2\mathrm{ext}}$ must come together at one point at the end of the core, i.e., these quantities must go to zero at the single value $x = x_z$.

To determine the second empirical constant, we compared the calculated width of the external and internal mixing zones with experiment, for a jet stream velocity ratio of $u_0/V_\infty = 4.83$. It turned out that the value of c_2 can be taken to be 0.1.

The value of b_{02} proved to be close to 0.5, i.e., the position of the axis at the nozzle lip is almost no different from the position in the ordinary symmetric jet.

Figure 3 shows the calculated widths of the external and internal mixing zones and their boundaries for $u_0/V_\infty = 3.23$, compared with experimental data of the author. Here the empirical constant c_1 was determined from the first equation of Eq. (16), with expansion of the mixing zone of the ordinary immersed jet flowing from the same nozzle, and the value obtained was $c_1 \sim 0.049$.

The position of the jet axis relative to which the external and internal mixing zones are located was calculated at each section at which we measured the total pressure, assuming that the radius of curvature of the jet axis is constant and that the jet passes through the middle of the nozzle at the nozzle lip.

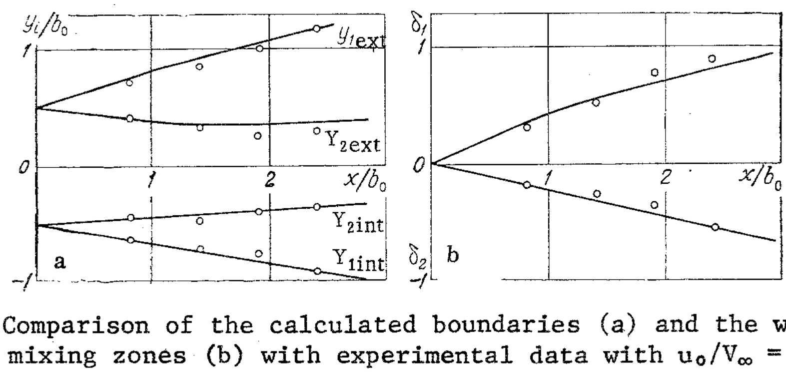

Fig. 3. Comparison of the calculated boundaries (a) and the widths of the the mixing zones (b) with experimental data with $u_0/V_\infty = 3.23$.

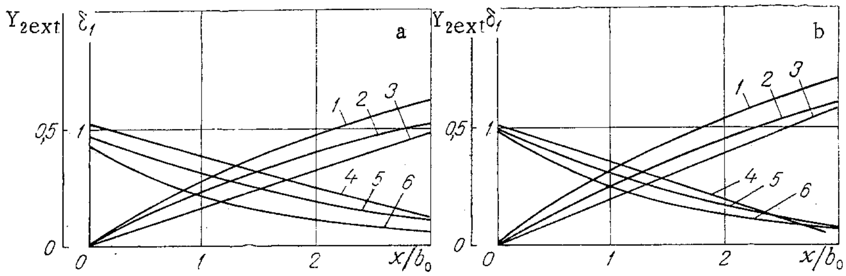

Fig. 4. Boundaries of the external mixing zone and its width as a function of distance from the nozzle lip and the velocity ratio of the carrier stream and the jet $m = V_\infty/u_0$: a) $c_1 = 0.04$; b) $c_1 = 0.05$; 1-3) δ_1; 4-6) y_{2ext}; 1) m = 0.30; 2) 0.20; 3) 0.05; 4) 0.05; 5) 0.20; 6) 0.30.

It can be seen from Fig. 4 that there is quite satisfactory agreement between the calculated and the experimental values. The width of the internal mixing zone can be considered to be the same as for an immersed jet.

To calculate the width and the upstream boundaries of the mixing zone for a given jet to stream velocity ratio u_0/V_∞ we can use the graph of Fig. 4, which shows δ_1 and y_{2ext} as a function of x, for various values of u_0/V_∞, c_1 and with $c_2 = 0.1$. From this graph we can determine the parameters of the upstream mixing zone of a planar jet in a carrier stream without the aid of a computer.

NOTATION

b_{02}, width of the downstream part of the jet at the nozzle lip (from the jet axis to the downstream lip of the nozzle); p, static pressure; R, radius of curvature of the jet axis; u_0, velocity at the downstream nozzle lip; u_1, velocity in the potential core of the jet; u_{1ext}, u_{1int}, velocity at the upstream and downstream boundaries of the jet; V_∞, velocity of the reference stream; y_{1ext}, y_{1int}, ordinates of the upstream and downstream boundaries of the jet; y_{2ext}, y_{2int}, ordinates of the upstream and downstream boundaries of the potential core; x, y) curvilinear coordinates; x, coordinate directed along the jet axis; y, coordinate perpendicular to x; α, angle between the jet axis and the direction of the undisturbed carrier stream; δ_1, δ_2, width of the upstream and downstream mixing zones; $\eta_{ext} = (y_{1ext} - y)/\delta_1$; $\eta_{int} = (y_{1int} - y)/\delta_2$; ξ, ζ, rectangular coordinates fixed in the nozzle — ξ is directed along the reference stream, and ζ is perpendicular to ξ.

LITERATURE CITED

1. T. A. Girshovich, "Theoretical and experimental investigation of a planar turbulent jet in a carrier stream," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 5, 121 (1966).
2. K. K. Fedyaevskii, A. S. Ginevskii, and A. V. Kolesnikov, Calculation of the Turbulent Boundary Layer of Incompressible Fluid [in Russian], Sudostroenie, Leningrad (1973).
3. B. R. Morton, "On a momentum mass flux diagram for turbulent jets, plumes, and wakes," J. Fluid Mech., 10, No. 1, 101 (1967).
4. T. A. Girshovich, "Calculation of the parameters of a planar turbulent jet in a carrier stream," Inzh. -Fiz. Zh., 25, No. 5, 907 (1973).
5. E. V. Bruyatskii, "An integral method of computing the entrance section of a planar turbulent jet in a carrier stream," Prikl. Mekh., 14, No. 3, 114 (1978).
6. T. A. Girshovich, "The turbulent jet in a carrier stream," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1, 151 (1966).